THERMAL SELF-FOCUSING OF ELECTROMAGNETIC WAVES IN A COMPLETELY IONIZED PLASMA

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Self-focusing effects can be important in propagation of electromagnetic waves in plasma. One of the possible mechanisms for self-focusing (thermal) is manifested over a wide range of plasma parameters, encompassing both laboratory and ionospheric or cosmic plasma. In this case, as experiments show, thermal self-focusing can appear at radiation powers of about 100 watts. A large number of papers is devoted to an analysis of thermal self-focusing (see, for example, [1-5]).

In this paper, we examine thermal self-focusing of electromagnetic waves in the millimeter range in a completely ionized plasma. If we limit ourselves to pulses with microsecond duration τ and characteristic transverse sizes r_0 in the range 1-10 cm, then it is possible to use an average interaction of the electromagnetic field with the plasma. It is assumed that the condition for quasilinearity of the plasma is satisfied and that the absorption of energy in the field by the plasma stems from simple motion of electrons in the field of the wave without taking into account turbulence, decay process, and so on, while energy is exchanged by plasma components due to Coulomb collisions. These assumptions are valid in the experimental situations mentioned above.

On the strength of this, we will use a hydrodynamic description of the plasma motion. Evidently, in this case, it is possible to neglect the motion of the plasma along the axis of the pulse.

In most theoretical papers on thermal self-focusing, it is assumed that the deviations of the plasma parameters from their equilibrium values are small. Then, it is possible to use the linearized equations of hydrodynamics. However, often such an approach is not adequate and, therefore, it is necessary to examine a complete system of equations of hydrodynamics. An example is the problem related to uhf heating of a plasma, where the plasma parameters can greatly deviate from their equilibrium values.

Under typical experimental conditions, i.e., plasma density n of the order of 10^{14}-10^{15} cm^{-3} and plasma temperature T in the interval 0.1-1 eV, the electron–ion collision frequency ν_{ei} is of the order of 10^{10} sec^{-1}. Therefore, the characteristic time for equalization of the temperatures of plasma components, equal to $(M/m)/\nu_{ei}$, where M is the mass of an ion and m is the mass of an electron, is 10^{-7}-10^{-8} sec, which is much less than the duration of the pulse. It follows from this that the temperatures of the plasma components can be assumed to be identical. For a similar reason, it is possible to neglect the ionic thermal conductivity, since under the conditions indicated the characteristic time for it is 10^{-2}-10^{-3} sec.

The complete system of equations that describes thermal self-focusing under these conditions, for axially symmetric beams, has the form

$$\frac{\partial n}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r} rnv = 0, \quad \frac{\partial nv}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r} rnv^2 = -\frac{1}{M}\frac{\partial p}{\partial r},$$
$$\frac{\partial W}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r} r(W+p) = \frac{e^2 n}{m}\frac{\tau_c |E|^2}{1+\omega^2\tau_c^2} + \frac{5}{r}\frac{\partial}{\partial r} r\left(\frac{nT}{m\nu_{ei}}\frac{\partial T}{\partial r}\right), \tag{1}$$
$$2ik\frac{\partial E}{\partial z} + \frac{1}{r}\frac{\partial}{\partial r} r\frac{\partial E}{\partial r} + \frac{4\pi e^2}{mc^2}\omega\tau_c\frac{i-\omega\tau_c}{1+\omega^2\tau_c^2} nE = 0.$$

Here, the z axis is directed along the pulse axis; $W = 3nT + (1/2)(M+m)v^2 n$; v is the radial velocity of the plasma; e is the charge of an electron; ω and E are the frequency and amplitude of the wave field, so that the intensity of the electric field is $\mathcal{E} = (1/\sqrt{2})(Ee^{i\omega t} + E^* e^{-i\omega t})$; $k = \omega/c$; $p = nT$ is the plasma pressure; τ_c is the average free flight time of electrons in the plasma.

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We note that in these equations the electrostriction pressure

$$p_{st} = \frac{1}{8\pi}\left(\frac{\omega_p}{\omega}\right)^2 |E|^2,$$

where ω_p is the plasma frequency, is not taken into account.

The ratio of the high-frequency pressure force $\nabla p_{st} \sim p_{st}/r_0$ to the thermal pressure force $\nabla p_T \sim p_T/r_T$ is

$$\frac{\nabla p_{st}}{\nabla p_T} \sim \frac{\frac{1}{8\pi}\left(\frac{\omega_p}{\omega}\right)^2 |E|^2/r_0}{\frac{1}{4\pi}\left(\frac{\omega_p}{\omega}\right)^2 \nu_{ei}\tau |E|^2/r_T} \sim \frac{r_T}{r_0\nu_{ei}\tau} \sim \frac{1+\frac{v\tau}{r_0}}{\nu_{ei}\tau},$$

where $p_T \sim \frac{1}{4\pi}\left(\frac{\omega_p}{\omega}\right)^2 \nu_{ei}\tau|E|^2$; and $r_T \sim r_0 + v\tau$ is the characteristic scale of the temperature nonuniformity. It follows from here that it is important to take into account striction when $\nu_{ei}\tau \lesssim 1$, i.e., for a collisionless plasma and for $v/r_0\nu_{ei} \gtrsim 1$.

For the plasma parameters and fields examined here ($n \sim 10^{14}$ cm^{-3}, $T \sim 0.1$-1 eV, $\tau \sim 10^{-6}$ sec, $r_0 \sim 1$ cm), the quantities ν_{ei} and v have the following order of magnitude: $\nu_{ei} \sim 10^{10}$ sec^{-1}, $v \lesssim 10^5$-10^6 cm·sec^{-1}, when $\nu_{ei}\tau \gg 1$ and $v/r_0\nu_{ei} \ll 1$ and, therefore, it is not important to take into account striction. However, as the temperature increases, the contribution of striction will increase rapidly and already for $T \sim 5$-10 eV the striction pressure will become dominant, since the ratio v/ν_{ei} increases like $T^{5/2}$.

At the initial stage of self-focusing, the deviations n' and T' from the equilibrium values n_0 and T_0 are small. Then, we obtain from (1), in the first approximation, the system of equations

$$\frac{\partial^2 n'}{\partial t^2} - c_s^2\frac{1}{r}\frac{\partial}{\partial r}r\frac{\partial n'}{\partial r} = \frac{n_0}{M}\frac{1}{r}\frac{\partial}{\partial r}r\frac{\partial T'}{\partial r},$$

$$\frac{\partial T'}{\partial t} - \frac{\varkappa}{3n_0}\frac{1}{r}\frac{\partial}{\partial r}r\frac{\partial T'}{\partial r} = \frac{e^2}{3m}\frac{\tau_c|E|^2}{1+\omega^2\tau_c^2}, \tag{2}$$

$$2ik\frac{\partial E}{\partial z} + \frac{1}{r}\frac{\partial}{\partial r}r\frac{\partial E}{\partial r} + \frac{4\pi e^2}{mc^2}\left(\frac{i\omega\tau_c n_0}{1+\omega^2\tau_c^2} - \frac{\omega^2\tau_c^2}{1+\omega^2\tau_c^2}n'\right)E = 0,$$

where $c_s = (T/M)^{1/2}$ is the velocity of sound and $\varkappa$ is the coefficient of thermal conductivity of the plasma.

The behavior of the solutions of system (2) depends on the quantities c_s and $\varkappa$. For small c_s and $\varkappa$, i.e., $c_s \ll r_0/\tau$ and $\varkappa \ll r_0^2 n_0/r$, as shown in [6], self-focusing is accompanied by strong self-modulation of the pulse. In this case, the maximum attainable pulse amplitude increases as the duration of the starting pulse approaches some critical value τ_0 like $(\tau_0 - \tau)^{-3/2}$.

Let us now examine the system (2) without assuming that n' and T' are small. Using the parabolic approximation, i.e., setting

$$E = \psi(t)\exp\left(-r^2/r_0^2F^2\right)/F,$$
$$n' = N(z)g\exp\left(-r^2g/r_0^2\right),$$

we obtain equations for the functions F(z, t) and g(z, t)

$$\frac{\partial^2 g}{\partial t^2} + 4c_s^2 g^2 = \beta\int^t \psi^2(t')\frac{dt'}{F^4}, \quad \frac{\partial^2 F}{\partial z^2} - 2\left(\frac{4}{r_0^4F^4} - \gamma N g^2\right)F = 0, \tag{3}$$

where β and γ are some constants.

For large amplitudes 1/F and g, if it is assumed that these quantities increase slowly as functions of their arguments, we can neglect the derivatives in (3). Then we obtain the estimate

$$g^2 = \frac{\gamma\beta N r_0^4}{16c_s^2}\int^t \psi^2(t')g^2(t')\,dt \quad \text{for} \quad \psi = \text{const},$$
$$g^2 \sim a_1 + \exp\left(a_2 t/c_s^2\right),$$

where a_1 and a_2 are constants.

Thus, at the initial stage of self-focusing, we can expect exponential growth in the field maximum and perturbation of the density with time. The growth increment in this case increases with decreasing sound velocity. On the whole, the picture is close to that for self-focusing of pulses in media with a Kerr nonlinearity (see [6]).

In order to study the nonlinear stage of self-focusing, it is necessary to solve the system (1) numerically. However, certain conclusions can be drawn by leaving the linear approximation. In the linear approximation, the coefficient of absorption is assumed to be constant, but in actuality, its magnitude is proportional to the density and decreases with temperature like $T^{3/2}$. Since in the process of self-focusing, the density drops, while the temperature decreases, the coefficient of absorption must decrease. This, in its turn, slows down self-focusing and must lead to its saturation.

Let us estimate the range of temperatures in which thermal self-focusing can exist. Evidently, this range determines both the temperature of the initial plasma, for which self-focusing is possible, as well as the maximum attainable temperature in the self-focusing process.

An estimate for the density perturbation follows from the equation for the field in (1)

$$n' > mc^2/4\pi e^2 r_0^2. \tag{4}$$

From the second equation of the same system, we obtain

$$n' \sim n_0 T' \tau^2 / M r_0^2,$$

while from the third equation, we have the estimates

$$T' \sim \frac{e^2 n}{\alpha m \omega^2} |E|^2 T^{-3/2} \tau, \quad \alpha = \frac{m^{1/2}}{4\pi e^4 \ln \Lambda}. \tag{5}$$

Substituting the two last estimates into (4), we obtain

$$T^{3/2} < \left(\frac{\omega_p}{\omega}\right)^2 \frac{\omega_p^2 |E|^2 \tau^3}{4\pi M c^2 \alpha}. \tag{6}$$

On the other hand, at the developed stage of self-focusing, when $T' > T_0$ and, therefore, $T \approx T'$, we obtain from (5)

$$T > Mc^2/\tau^2 \omega_p^2. \tag{7}$$

Inequalities (6) and (7) give the approximate boundaries for the temperature range sought.

For convenience in constructing a numerical model, we transform to dimensionless variables in (1). If t, r, z, n, v, T, and E are measured in units τ_0, r_0, kr_0^2, $v_0 = c_S^2 \tau / r_0$, T_0 and E_0, respectively, then, taking into account the inequality $\omega^2 \tau^2 \gg 1$, we obtain

$$\frac{\partial n}{\partial t} + \frac{\nu}{r}\frac{\partial}{\partial r} rnv = 0, \quad \frac{\partial nv}{\partial t} + \frac{\nu}{r}\frac{\partial}{\partial r} rnv^2 = -\mu \frac{\partial}{\partial r} nT,$$

$$\frac{\partial}{\partial t}(nT + \lambda nv^2) + \frac{\nu}{r}\frac{\partial}{\partial r} rv\left(\frac{5}{3} nT + \lambda nv^2\right) = \frac{\varkappa}{r}\frac{\partial}{\partial r} rT^{5/2}\frac{\partial T}{\partial r} + \chi \frac{n^2 |E|^2}{\eta^2 T^{3/2}}, \tag{8}$$

$$2i\frac{\partial E}{\partial z} + \frac{1}{r}\frac{\partial}{\partial r} r \frac{\partial E}{\partial r} + \sigma\left(\frac{in^2}{\eta^2 T^{3/2}} - \frac{n}{\eta}\right) E = 0,$$

where $\nu = \tau_0 v_0 / r_0$; $\mu = 2T_0\tau_0 / Mv_0 r_0 = 2$; $\lambda = Mv_0^2 / 6T_0$; $\eta = \alpha\omega^2 T_0^{3/2} / n_0$; $\chi = \frac{2}{3}\tau_0 \alpha e^2 T_0^{1/2} |E|^2 / n_0$; $\sigma = 4\pi\omega r_0^2 \alpha e^2 T_0^{3/2} / mc^2$; and $\varkappa = \frac{5\tau_0 \alpha T_0^{5/2}}{3mn_0 r_0^2}$ are dimensionless quantities.

Within the scope of system (8), we modelled the propagation of pulses having an initial profile of the form

$$E = A \exp\left(-r^2/r_0^2 - t^2/t_0^2\right),$$

with the following base values of the parameters: $\nu = 0.2$, $\mu = 2$, $\lambda = 0$, $\sigma = 300$, and $\eta = 25$. These parameters correspond to a hydrogen plasma with an initial temperature of about 0.1 eV, density 10^{14} cm^{-3}, pulse duration $0.2 \cdot 10^{-6}$ sec, and pulse width 0.3 cm with the incident radiation frequency $4 \cdot 10^{12}$ sec^{-1}.

In the numerical modelling, the initial temperature of the plasma and the incident pulse energy were varied.

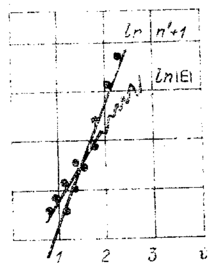

Fig. 1

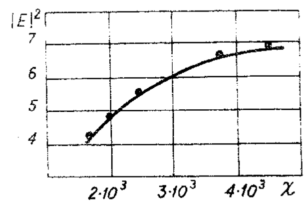

Fig. 2

Figure 1 illustrates the exponential growth at the field maximum $E_m = \max E$ and maximum density perturbation $n'_m = \max n'$ along the pulse axis at the initial stage of self-focusing, following from an analysis of the linearized system (2). As is evident from numerical experiments, the conclusion drawn is quite well confirmed, if c_s is sufficiently large; i.e., $c_s \tau / r_0 > 1$. At later stages of self-focusing, the rate of growth of E_m and n'_m slows down and oscillations even appear.

Figure 2 shows the dependence of the maximum field amplitude attainable with self-focusing $E_{max} = \max |E|^2$ on the energy of the initial pulse. The effects of self-focusing appear beginning with an amplitude in the initial pulse of 0.5 cgse. As the starting amplitude increases, the maximum intensity increases quite rapidly to some level, which in our calculations exceeded the initial value by a factor of 6–8. After attaining the maximum, the field amplitude stabilized at a level of about 1.5 cgse. Approximately this picture was observed experimentally in [2].

Until saturation is achieved, the pulse profile remains smooth. If the maximum of the amplitude attained does not fall into the saturation region, then after the maximum is attained the pulse spreads out and is absorbed by the plasma.

If, on the other hand, the power in the pulse is such that saturation of self-focusing is attained, then its evolution is of a more complicated nature. After the maximum amplitude is attained, the pulse spreads out as before, but in contrast to the case of low powers, in this case, a new maximum appears and begins to grow in front of the field maximum. Energy flows into the region of the new peak. After attaining some level, a new peak arises in front of this peak and the picture is repeated, while there is still energy in the pulse. Thus, for sufficiently high pulse power, its evolution represents a sequence of pulsations-spikes that gradually decay due to absorption. We note that these peaks in the amplitude remain practically stationary in space, i.e., stationary luminous points, standing foci, are formed on the pulse axis.

The typical evolution of the axial profile of a pulse after attaining the first maximum is shown in Fig. 3.

These results can be interpreted as follows: in the vicinity of the peak in the amplitude, the temperature greatly increases and the plasma density drops. In the calculations presented, the temperature on axis increased by a factor of 4–6, but the plasma density decreased by a approximately a factor of 2. As a whole, this leads to a decrease in the absorption coefficient of the plasma (approximately by a factor of 40 for the changes in the parameters indicated above), i.e., to saturation of the nonlinearity. Energy flows, being weakly absorbed, through the region with high transparency formed in the plasma and, if enough of it is left, a new peak can form somewhere in front. The evolution of the plasma temperature on axis is presented in Fig. 4.

Let us consider the role of the electronic thermal conductivity. For the parameters that we have chosen, the dimensionless coefficient of thermal conductivity is about 0.01. The introduction of such a coefficient of thermal conductivity into the equations leads to practically analogous results, having an insignificant effect near the temperature maximum. However, as the initial temperature is increased by a factor of 1.5, the effective thermal conductivity turned out to be significant, since the coefficient of electronic thermal conductivity increases with temperature like $T^{5/2}$. It is the increase in the absorption coefficient at high temperatures that led to saturation in self focusing in this case.

The nonlocal nature of the distribution of temperature and density did not have a significant effect on saturation, which arose at the leading edge of the pulse, since in this region the scale of the temperature nonuniformity $r_T \sim r_0 + vt$ and that of density $r_n \sim r_0 + (v + c_s)t$ are of the order of the transverse size of the pulse r_0. However, for large t and small z, i.e., on the trailing edge of the pulse (see Fig. 3), the scale of the nonuniformity r_n begins to exceed the size of the pulse, diffraction spreading occurs, and self-focusing ceases.

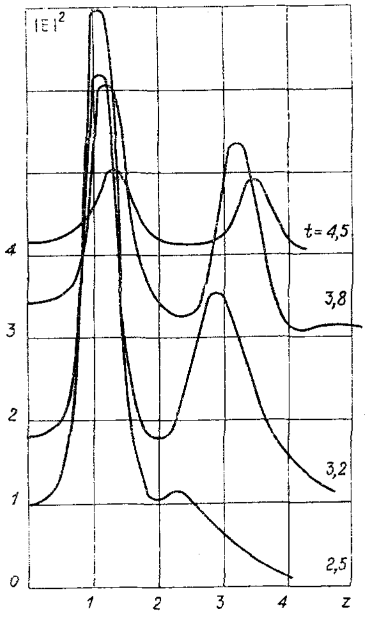

Fig. 3

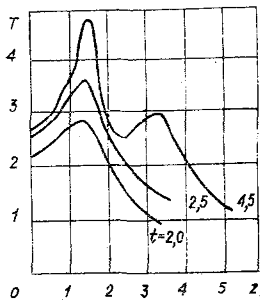

Fig. 4

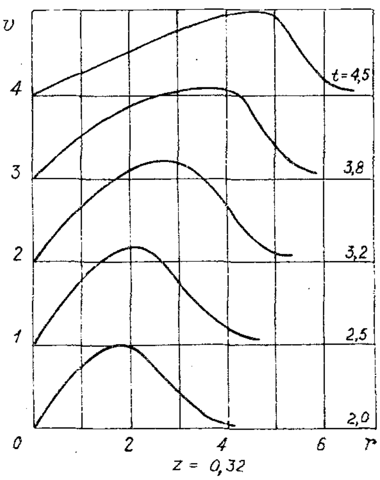

Fig. 5

We note that at the developed stage of self-focusing, there arises strong convective motion of plasma out of the near-axial region, and a velocity profile close to a shock profile forms, as shown in Fig. 5.

The results of the numerical modelling presented above also encompass the range of variation of parameters of the ionospheric plasma, for which self-focusing of waves in the meter range was observed in [7, 8].

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